

# Proposal for witnessing non-classical light with the human eye

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**We give a complete proposal showing how to detect the non-classical nature of photonic states with naked eyes as detectors. The enabling technology is a sub-poissonian photonic state that is obtained from single photons, displacement operations in phase space and basic non-photon number resolving detectors. We present a detailed statistical analysis of our proposal including imperfect photon creation and detection and a realistic model of the human eye. We conclude that a few tens of hours are sufficient to certify non-classical light with the human eye with a p-value of 10%.**

## 1 Introduction & motivations

A lot of efforts have been recently devoted to lay the theoretical groundwork needed to realize the first quantum experiment with the human eye. These proposals include the detection of entanglement through Bell tests [1–3] and entanglement witnesses [4]. Ref. [4], in particular, uses displacement operations in phase space to upgrade the eye up to the point where it can reveal path entanglement, *i.e.* entanglement between two modes sharing a single photon. As path entanglement and displacement operations are routinely realized in practice, this proposal might lead to the first experiment where entanglement is detected with the human eye.

The motivations for such an experiment are manyfold. First, it is a fascinating attempt to get closer to the quantum world. Indeed, it is conceptually very different from standard quantum optics experiments where measurements are done by photon detectors and the sole role of experimentalists in the measurement process is to analyse the experimental data stored on a computer. The envisioned experiment is unitary until the eye, so if a collapse happens it does not happen before the eye. Second, while well integrated sources working at room temperature are available in quantum optics experiments, intensive efforts are being made to develop efficient, noiseless and easy-to-use photon detectors working at room

temperature [5]. These efforts, however, are carried out at the material level and the technology is becoming complex. Proving that the quantum optics toolbox such as displacement in phase space can be used to upgrade rudimentary detectors like the human eye up to the point where they can be used for quantum optics experiments is a new paradigm for applied physics. Third, such an experiment interfaces quantum light and biological systems. Inspired by the great success of quantum optics in revolutionizing communications [6], metrology [7], sensing [8] or computing [9], this experiment of a new kind may flourish with important applications for biomedical research.

These motivations naturally raise the question of the simplest quantum experiment which one can envision with the human eye as a detector. As stated before, the proposal of Ref. [4] is appealing as it uses simple ingredients, namely single photon path entanglement and displacement operations. In this manuscript, we derive a witness for non-classical states and we show how the same ingredients allow one to reveal the non-classical nature of a single photon in superposition with vacuum. While entanglement detection requires measurements in different bases, the experiment that we propose is simpler as it uses displacement operations with fixed amplitudes and phases. It does not need interferometric stabilization of optical paths and is very robust against loss. We show through a detailed feasibility study including a realistic model of the human eye with a reasonable recovery time as well as imperfect photon creation and detection, that a few tens of hours are sufficient for our witness to conclude about non-classicality with a p-value of 10%. Our results point towards a ready-to-use proposal for implementing the first quantum experiment with the human eye.

## 2 Witnessing non-classicality with rudimentary detectors

Coherent states  $|\alpha\rangle$  of an harmonic oscillator (or a mode of the electromagnetic field) saturate the uncertainty relations for any pair of quadratures as well as for amplitude and phase [10]. In addition, they are eigenstates of the positive frequency part of the quantized field and vector potential operators [11]. For these reasons, the set of coherent states is thought as the most classical subset of all possible pure states of light. In this context, a state which can be expressed as a mixture of coherent states  $|\alpha\rangle$

$$\rho_{\text{class}} = \int d^2\alpha p(\alpha) |\alpha\rangle\langle\alpha|, \text{ with } p(\alpha) \geq 0 \quad (1)$$

is considered classical, and any state which cannot be decomposed in this way is then non-classical. It is easy to see that the convex combination of coherent states in Eq. (1) satisfies  $(\langle\hat{N}^2\rangle - \langle\hat{N}\rangle)/\langle\hat{N}\rangle^2 \geq 1$  with  $\hat{N}$  the number operator [12]. Hence, a photon counting detector can be used to witness the non-classical nature of a light state. If the photon counting results reveal  $(\langle\hat{N}^2\rangle - \langle\hat{N}\rangle)/\langle\hat{N}\rangle^2 < 1$ , we can indeed conclude that the measured state is non-classical. Note that all non-classical states lead to entanglement when combined with the vacuum on a beamsplitter [13]. The link with entanglement helps clarifying the notion of non-classical states.

Moreover for few photon states,  $(\langle\hat{N}^2\rangle - \langle\hat{N}\rangle)$  can be approximated by  $\sim 2\langle|2\rangle\langle 2|\rangle$  and  $\langle\hat{N}\rangle^2$  by  $\sim (\langle|1\rangle\langle 1|\rangle)^2$ . Hence, one can use a 50/50 beamsplitter and two non photon number resolving detectors to witness the non-classical nature of few photon states by checking that the two-fold coincidence ( $\sim \langle|2\rangle\langle 2|\rangle/2$ ) are smaller than the product of singles ( $\sim \langle|1\rangle\langle 1|\rangle^2/4$ ), cf. [14] for a proper derivation. Can one still use this criterion in presence of other kinds of detectors? We now address the question of the conditions required to witness the non-classical nature of a light source with a 50/50 beamsplitter and two detectors.

Let us consider an arbitrary detector with a binary outcome, one corresponding to click, the other one to no-click. We label  $p_s(\alpha)$  the probability to get a click when impinging a coherent state  $|\alpha\rangle$  on such a detector. In a scenario where two of these detectors are placed after a 50/50 beamsplitter, the probability to get a twofold coincidence with any classical state is given by  $p_c(\rho_{\text{class}}) = \int d^2\alpha p(\alpha) p_s(\alpha/\sqrt{2})^2$  whereas the probability of a single detection is given by  $p_s(\rho_{\text{class}}) = \int d^2\alpha p(\alpha) p_s(\alpha/\sqrt{2})$ . This simply comes from the fact that a coherent state splits into two similar coherent states on a beamsplitter  $|\alpha\rangle \xrightarrow{BS} |\frac{\alpha}{\sqrt{2}}\rangle_t |\frac{\alpha}{\sqrt{2}}\rangle_r$ . The Cauchy-Schwarz inequality  $\int f(\mu)^2 d\mu \int g(\mu)^2 d\mu \geq (\int f(\mu)g(\mu) d\mu)^2$  for  $f = 1$ ,

$g = p_s(\alpha/\sqrt{2})$  and  $d\mu = p(\alpha)d^2\alpha$  then implies

$$\frac{p_c(\rho_{\text{class}})}{p_s(\rho_{\text{class}})^2} \geq 1. \quad (2)$$

In other words, any detector can be used to witness non-classicality as long as one has two copies of this particular detector. It suffices to place these detectors after a 50/50 beamsplitter and to record the number of singles and coincidences. If the ratio between the probability of having a coincidence and the square of the probability of singles is smaller than one, we can safely conclude that the measured state is non-classical. We show in the appendix A that the ratio between the coincidence and the product of singles is a witness for non-classicality even if the two detectors after the beamsplitter are not identical and the beamsplitter is not balanced, as long as  $p_s(\alpha)$  is an increasing function of the photon number  $|\alpha|^2$  for both detectors. These results are used in the next section to show how to detect non-classical states with the human eye.

## 3 Witnessing non-classicality with the human eye

Let us start this section by recalling how to model the response of the human eye to weak light stimuli. In a landmark experiment Hecht, Schlaer and Pirenne tested the capability of the human eye to detect light pulses containing only a few photons [15], see also [16] for a review on the sensitivity of the human eye. In their experiment, an observer was presented with a series of coherent light pulses and asked to report when the pulse is seen. The results of this experiment are very well reproduced by a model involving a threshold detector preceded by loss. In particular, the experimental data is compatible with a threshold at  $\theta = 7$  photons and an efficiency of  $\eta_e = 8\%$  [2, 4]. Note that the results of Hecht *et al.* have been reproduced recently in an experiment aiming to test the ultimate limit of the human eye [17].

Given the witness for non-classical states presented in the previous section, we envision an experiment where two eyes are placed after a beamsplitter. The event “click” corresponds to the case where the observer sees light, “no-click” when no light is seen. The experiment is repeated several times to access the probability to see light with one of the two eyes as well as the joint probability to see light with both eyes. The ratio between the coincidences and the product of singles is then used to reveal non-classicality. This ratio is labeled  $g^{(2)}(0)$  in analogy to the standard autocorrelation measurement.

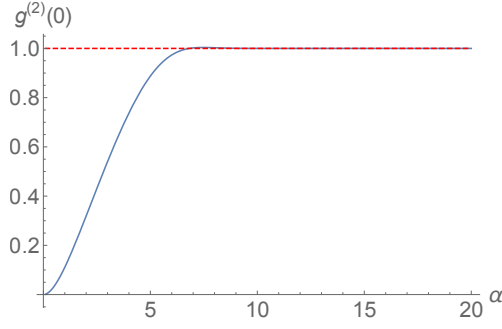


Figure 1: Result of an auto-correlation ( $g^{(2)}(0)$ ) measurement in which two eyes are placed after a 50/50 beamsplitter. The ratio between the probability to see light with both eyes and the square of the probability to see light with one eye is recorded for an input state  $\mathcal{D}(\alpha)(|0\rangle + |1\rangle)/\sqrt{2}$  considering real  $\alpha$ . We here show this ratio as a function of  $\alpha$ . Ratio smaller than one witnesses the non-classical nature of the state.

To make a complete proposal, we still need to find a quantum state for which the non-classical nature can be revealed in such a setup. Note that sub-poissonian states, *i.e.* states for which the distribution in photon number space is narrower than the one of a coherent state with the same mean photon number, are natural candidates for achieving  $g^{(2)}(0) < 1$  with threshold detectors, such as the human eye. This is because there is a regime where, for the same probability of singles, the narrow photon number distribution of a sub-poissonian state yields a lower coincidence probability than the one of the corresponding coherent states. As an illustration, consider an ideal threshold detector and a Fock state that has enough photons to eventually make one of the detectors click, but not enough to give a coincidence.

While Fock states with large photon numbers are challenging to produce, a sub-poissonian state can be obtained in practice by displacing a superposition of vacuum and single photon Fock state in phase space. The resulting state  $\mathcal{D}(\alpha)\left|\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right\rangle$ , where  $\mathcal{D}(\alpha)$  stands for a displacement operation, indeed has a variance in photon number space that is  $\frac{1+8|\alpha|^2-4\text{Re}(\alpha)^2}{2+4|\alpha|^2+4\text{Re}(\alpha)}$  times that of a coherent state with the same mean photon number. This ratio admits values that are below one, and interestingly, for a given strength of the displacement  $|\alpha|^2$ , it is minimal and always inferior to unity when  $\alpha$  is real. Consequently, from here on we will only consider real displacements.

Fig. 1 shows the value of  $g^{(2)}(0)$  obtained when sending such a state on a 50/50 beamsplitter followed by two eyes as a function of the amplitude of  $\alpha$ . We see that the non-classical nature of  $\mathcal{D}(\alpha)\left|\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right\rangle$  can be detected with the human eye as long as  $\alpha \leq 13.3$ . For larger  $\alpha$ , the two eyes always see light and the ratio between coincidences and singles tends to one. However, in the range of displacement values  $\alpha \sim 10$ ,

one can expect non-negligible occurrence frequency for the event “seen” for both eyes. These encouraging estimations compel us to make a detailed feasibility study, *i.e.* to propose a practical way to create a single photon superposed with vacuum, to account for imperfect generation efficiency, channel loss, limited detection efficiencies and to conclude about the statistics that is required to witness non-classicality with the human eye.

## 4 Proposed experiment

The experiment we envision is shown in Fig. 2. A source based on spontaneous parametric down conversion is used to create photon pairs, the detection (on detector  $D_h$  in Fig. 2) of one photon from a given pair serving to herald the presence of its twin. The latter is then sent into a 50/50 beamsplitter to create path-entanglement, *i.e.* entanglement of the form  $(|0\rangle_t|1\rangle_r - |1\rangle_t|0\rangle_r)/\sqrt{2}$  between the transmitted and reflected modes of the beamsplitter which share a single photon. The reflected mode is subsequently detected with a non-photon number resolving detector (detector  $D_g$  in Fig. 2) preceded by a displacement in phase space  $\mathcal{D}(\beta)$ . With the appropriate displacement amplitude, such a measurement performs a pretty good measurement along the  $x$  direction of the Bloch sphere having  $|0\rangle$  and  $|1\rangle$  as its north and south pole respectively [18]. In other words, with the appropriate displacement, a detection click projects the transmitted mode into a state close to  $(|0\rangle_t + |1\rangle_t)/\sqrt{2}$ . Such a state is then displaced in phase space, split using a 50/50 beamsplitter and sent to human observers. The single and coincidence events are recorded and the experiment is repeated until the observers can conclude about the non-classical nature of the superposition state with enough statistical confidence.

Note that in this setup, one can tune the transmission coefficient of the first beamsplitter along with the displacement amplitude  $\beta$ , effectively modifying the input state for the autocorrelation measurement. Finally, we observed that the closest state to  $\mathcal{D}(\alpha)\left|\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right\rangle$  is obtained by choosing a highly unbalanced beamsplitter with transmission  $t \sim 1$  and using a displacement  $\mathcal{D}(\beta)$  with almost zero amplitude. In this case, we get a very partially entangled state and maximum coherence of the conditional state  $(|0\rangle_t + |1\rangle_t)/\sqrt{2}$  is restored by measuring the reflected mode almost along the  $z$  direction and post-selecting the case where a click is obtained. This favors larger fidelities of the conditional state because the measurement noise is reduced when it gets closer to the  $z$  direction [18]. However, the probability to get a click drops when the transmission of the beam-

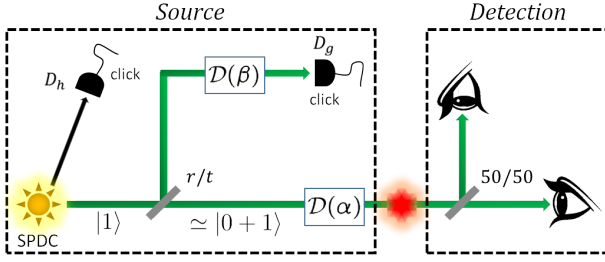


Figure 2: Scheme of the envisioned setup aiming to witness the non-classical nature of a superposition state  $\mathcal{D}(\alpha)(|0\rangle_t + |1\rangle_t)/\sqrt{2}$  with the human eye. The superposition  $(|0\rangle_t + |1\rangle_t)/\sqrt{2}$  is prepared by first sending a single photon into an unbalanced beam-splitter and by a subsequent detection of the reflected mode with photon detector preceded by a displacement operation. For displacement with a small enough amplitude, this projects the transmitted mode into a state close to the desired superposition. This superposition state is then displaced to produce the non-classical state of interest. A 50/50 beamsplitter and two eyes are then used to analyze this state with a measurement analog to an auto-correlation measurement.

splitter increases. There is thus a trade-off between the “quality” of the states produced by the source and the rate at which they are produced. The parameters  $\beta$  and  $t$  have to be optimized in view of the statistics needed to witness non-classicality, cf. below.

Several requirements need to be satisfied for implementing the experiment proposed in Fig. 2. (i) The efficient generation of pure, indistinguishable and narrowband single photons is the first one. A straightforward way to create photons with these properties from spontaneous parametric down conversion is to combine short, Fourier limited pump pulses with a narrow-band filtering of the heralding photons. This results in Fourier limited heralded photons with the spectrum of the pump [19]. To ensure a high coupling efficiency of these heralded photons into an optical fiber, a plane wave pump is required and the heralding photons need to be spatially filtered with a single mode fiber before being detected. This projects the heralded photons into the fundamental spatial mode of the fiber, and hence allows one to reach very high coupling efficiencies [20]. (ii) The photons need to have a color that can be seen by the human eye. This can be fulfilled with a pump at 405nm down converted into non-degenerate photon pairs at 1536 and 550nm. The advantage is threefold. 550nm is very well suited for the human eye and the photons in the telecom band can be efficiently filtered both spatially and in frequency. The telecom mode can also be seeded with a stable cw telecom laser to generate the coherent states that are needed for the

displacement operations, cf. below. (iii) The click rate on the detector  $D_g$  in Fig. 2 needs to be adapted to the timescale of the response of the human eye as it sets a start for the observer. This can be done by reducing the repetition rate of the pump laser with an optical chopper. The heralding rate on  $D_h$  and thus on  $D_g$ , can then be easily set by tuning the laser intensity and the duty cycle of the optical chopper, c.f. below. (iv) To implement the displacement operations, we need an unbalanced beamsplitter and coherent pulses with Poissonian photon distribution that are indistinguishable from the photons at 550 nm in all degrees of freedom. This can be done using difference frequency generation. More precisely, we propose to use a second non-linear crystal, identical to the first one and pumped by the same laser but with a narrow seed of the telecom mode. In contrast to spontaneous parametric down conversion, the seed results in coherent states at 550nm with the characteristics of the pump laser, *i.e.* Fourier limited coherent states with the spectrum of the pump [21]. Since the coherent states created in this way and the single photons at 550nm are generated from the same pump, their indistinguishability is insensitive to the pump fluctuations. Note also that with a ps pump, the effect of frequency fluctuations of the telecom laser is negligible. The slow fluctuations in intensity of the latter can be recorded and taken into account once the measurements are done. In the worst case, they can be monitored and corrected with a feedback loop. Albeit with different wavelengths, the proposed technique has already been used successfully in various experiments [21, 22].

Concretely, we envision an experiment where a Ti-Sa laser is doubled to create 2 – 3ps pulses at 405nm with a repetition rate of 80 MHz. These pulses are then used to pump two crystals in order to be down converted to 1536 and 550nm respectively. The first crystal will be used to create pure single photons at 550nm by picking up a single spatial and frequency mode of the photons at 1536nm with a monomode fiber and a narrowband spectral filter. Coherent states that are indistinguishable from the photons at 550 nm are generated by seeding the second crystal with a pulsed telecom laser. Let us emphasize that the critical point of this experimental implementation is the noise. In standard experiment, the noise is filtered out by analyzing the detection times to discriminate between true and false events. As the response of the human eye is not fast enough for such a temporal discrimination, we need to be sure that a limited number of undesired photons can reach the eye of the observer. First, we propose to decrease the repetition rate of the pump laser to 1.6 MHz using an optical chopper with a duty cycle of 0.02. By tuning the pump intensity to get a pair emission probability of  $0.8 \times 10^{-3}$  and considering a global detection efficiency of 0.08 for  $D_h$ ,



(i.e. a coupling efficiency of 0.8, a filter transmission of 0.4 and a raw detection efficiency of 0.25), we get an heralding rate on  $D_h$  of  $\sim 100$  Hz. Moreover, we consider a coupling efficiency of the heralded photon at 550nm of  $\eta_c = 0.8$  in agreement with the experimental results reported e.g. in Ref. [20]. The detection efficiency of the visible detector in the upper arm of Fig. 2 is assumed to be  $\eta_d = 0.5$  which is realistic even when including the transmission loss from the source to the detector and the inefficiencies of linear optical elements. We neglect mismatches in the indistinguishability of the photons and coherent states at 550nm which is well justified given the results of Ref. [21] where the visibility of the Hong-Ou-Mandel interference between a single photon and a coherent state created via identical crystals as described before, was only limited by the statistics of the coherent state. We set the transmission  $t = 98\%$  which, together with the value of the displacement  $\beta \sim 0.08$  chosen to minimize the total number of experimental runs (cf. below), ensures that 1% of the heralds on  $D_h$  lead to a click on  $D_g$ . Meanwhile the conditional state generated on the lower arm shows a near maximal 95% fidelity with respect to  $\mathcal{D}(\alpha) | \frac{1}{\sqrt{2}}(0+1) \rangle$ .

The dominant noise in this scenario comes from the coherent states that are used for the displacement operations. We propose to trigger the seed that is used to generate these coherent states on detections in  $D_h$ . In this case, the noise is  $\sim 100$  times greater than the signal. To reduce it further, a pulse picker is placed in front of the eyes which is triggered by detections on  $D_g$ . Considering an extinction ratio of 1:2000, we get a signal-to-noise ratio of  $\sim 20$ , which should be more than enough to perform the proposed measurement. Note that the pulse picker also filters out other sources of noise, including the spontaneous emission of the crystal used to generate single photons at 550 nm (that is negligible with respect to the noise due to coherent states).

## 5 Statistics

To conclude the feasibility analysis of the proposed experiment, we now turn to the question of statistics, and determine the number of runs needed to exclude the possibility that the observed finite statistics are the result of measurements on a classical state. This is a particularly relevant question in our case, as the repetition rates that can be attained with the human eye are much lower than the slowest commercial detectors. The statistical study that we describe in this section aims at estimating the time-resource that an experimenter would have to allocate to such an experiment for the efficiencies discussed in the previous section, depending on the accuracy he wants to achieve.

The statistical issue is essentially an estimation of

the odds of having  $g^{(2)}(0) < 1$  from a classical photon-number distribution. To answer this, we consider the multinomial probability

$$P(N_s, N_c) = p_c^{N_c} (p_s - p_c)^{N_s - N_c} (1 - p_s)^{N - N_s} \times \binom{N}{N_c, N_s - N_c, N - N_s} \quad (3)$$

of obtaining  $N_s$  singles and  $N_c$  coincidences out of  $N$  experimental runs from the knowledge of the single and coincidence probabilities in one round  $\{p_s, p_c\}$ . Note that we assume here that the single probability on each eye is identical, and that the runs are independently and identically distributed (i.i.d.). Further note that the form of the above distribution, whose natural variables are  $N_c$  and  $N_s - N_c$ , stresses the dependence of the events “single” and “coincidence”. Indeed we have defined a single on one arm regardless of the situation on the other arm, hence a coincidence is counted as a single as well. The outcome “single only” has an occurrence probability  $p_s - p_c$  as can be seen in the multinomial expression. Both the quantum scenario presented before, with  $\{p_s(\rho_q), p_c(\rho_q)\}$  depending on the non-classical state  $\rho_q$ , and the classical one with  $\{p_s(\rho_c), p_c(\rho_c)\}$  such that  $p_c(\rho_c) > p_s^2(\rho_c)$  give rise to a probability distribution that we label respectively by  $P^q(N_s, N_c)$  and  $P^c(N_s, N_c)$ .

We then choose an estimator  $\chi$  which is a function of the total number of singles  $N_s$  and coincidences  $N_c$  observed in  $N$  rounds of the experiment, cf. below. For a given  $N$ , this estimator takes the value  $\chi(N_s, N_c)$  with probabilities  $P^q(N_s, N_c)$  and  $P^c(N_s, N_c)$  for the quantum and classical scenarios. The probability of observing a value of  $\chi$  smaller than a given value  $\chi_0$  in the quantum (classical) case after  $N$  rounds is thus given by

$$P(\chi^{q/c} \leq \chi_0) = \sum_{N_s, N_c | \chi(N_s, N_c) \leq \chi_0} P^{q/c}(N_s, N_c). \quad (4)$$

On one side, the quantum distribution tells us what is the probability with which we can expect to observe (in a quantum experiment) a value of  $\chi$  smaller or equal to some value  $\chi_0$ . We write this probability

$$P_{\text{stop}} = P(\chi^q \leq \chi_0). \quad (5)$$

On the other side, the classical distribution allows us to define the p-value  $\epsilon$  associated with the rejection of the null hypothesis “the state is classical” once a value  $\chi_0$  is observed. This p-value is given by

$$\epsilon = \max_{p_c \geq p_s^2} P(\chi^c \leq \chi_0) \quad (6)$$

where the maximum is taken over all classical scenarios satisfying  $p_c(\rho_c) \geq p_s^2(\rho_c)$ . Alternatively, we can read the relation (6) as a definition of the critical value

of the estimator  $\chi_0$  which needs to be obtained in order to rule out all classical states with a confidence of  $1 - \epsilon$ . Choosing first the p-value, Eq. (6) gives  $\chi_0$  which can then be used to get the probability to stop at the  $N^{\text{th}}$  run using Eq. (5). The average number of runs that is needed to rule out classical states can finally be estimated as (cf. Appendix B)

$$\langle N \rangle \simeq \sum_{j \geq 0} \frac{n(2j+1)}{2} (P_{\text{stop}}(n(j+1)) - P_{\text{stop}}(nj)) \quad (7)$$

where  $n$  is a coarse-graining parameter used to make the computation faster.

The question at this stage is what is a good choice for the estimator. Let us consider the space of frequencies defined by  $(f_s^2, f_c) \equiv \left( \left( \frac{N_s}{N} \right)^2, \frac{N_c}{N} \right)$ . We choose a set of coordinates  $\{x, y\}$  to cancel the covariance and to equal the variances of  $P^q(N_s, N_c)$  in the  $x$  and  $y$  directions at first order in  $\frac{1}{N}$ . This is achieved by setting

$$\begin{cases} x = \sqrt{\frac{c}{b}} f_s^2 + \frac{d}{\sqrt{cb}} f_c \\ y = \sqrt{\frac{b}{c}} f_c \end{cases} \quad (8)$$

with

$$\begin{cases} b = \sqrt{\frac{(1-p_c(\rho_q))p_s(\rho_q)}{(1-p_s(\rho_q))p_c(\rho_q)}} - 1 \\ c = \frac{1-p_c(\rho_q)}{2p_s(\rho_q)(1-p_s(\rho_q))} \\ d = -1 \end{cases}.$$

The projection of  $P^q(N_s, N_c)$  in the  $x-y$  plane hence defines circular isolines, cf. Fig. 3 red isolines. The dashed black line in Fig. 3 distinguishes the frequencies coming from classical and non-classical states. In particular, the distributions with mean values lying on this boundary come from states with  $p_c(\rho_c) = p_s^2(\rho_c)$ , i.e. coherent states with various  $p_s$ . The classical scenario that best reproduce the quantum statistics is quite clearly a coherent state which minimizes the Euclidean distance to the quantum distribution, i.e. centered on the orthogonal projection of the quantum distribution onto the dashed black line of Fig. 3. Such a coherent states is associated with

$$p_s(\rho_c) = \sqrt{\frac{c(c+d)p_s(\rho_q)^2 + (d(c+d) + b^2)p_c(\rho_q)}{b^2 + (c+d)^2}}. \quad (9)$$

Calling  $(x'_0, y'_0)$  the center of the corresponding distribution  $P^c(N_s, N_c)$ , an estimator of the form

$$\chi = y' - y'_0 + a(x' - x'_0)^2, \quad (10)$$

where  $x' = \cos(\phi)x + \sin(\phi)y$ ,  $y' = \cos(\phi)y - \sin(\phi)x$  and  $\phi = \arccos \frac{c+d}{\sqrt{b^2+(c+d)^2}}$  is intuitively minimized by the coherent state satisfying (9) for appropriate  $a$  as  $\phi$  is such that the axis of the parabola is orthogonal to the classical/non-classical boundary.

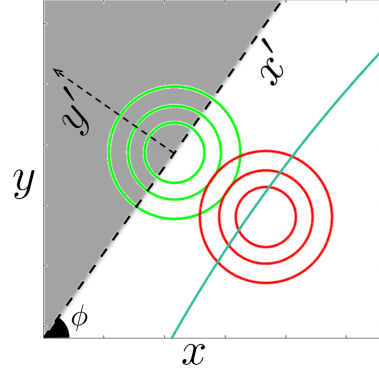


Figure 3: Projections in the modified frequency plane  $\{x, y\}$  defined in Eq. (8) of the probability distributions  $P^q(N_s, N_c)$  for the quantum scenario presented in Fig. 2 (red isolines) and  $P^c(N_s, N_c)$  for the coherent state defined in 9 (green isoline). The blueish contour line is the estimator given in Eq. (10). The dashed black line separates the mean values of quantum and classical states as witnessed by a  $g^{(2)}(0)$  measurement. In particular, the shaded area includes all states with  $g^{(2)}(0) \geq 1$ .

The probability that enough statistics is obtained after  $N$  runs to exclude the classical distribution  $P^c(N_s, N_c)$  with the estimator given in Eq. (10) can be computed numerically as a function of the steepness of the parabola  $a$  and the amplitude of displacement operations  $\alpha, \beta$ . After checking that the considered classical strategy is indeed optimal for the estimator (10), we obtained the optimal values  $a = 40$  and  $(\alpha, \beta) \simeq (10.99, 0.08)$  for the efficiencies discussed in the previous section. The results are shown in Fig. 4 for p-values of 1% and 10%. We see for example that after 350000 runs, we have more than 50% chance of being able to rule out classical states with a confidence of  $1 - \epsilon = 99\%$ . For  $n = 12500$ , we find  $\langle N \rangle \simeq 402964$  for a confidence of 99%. Note that to perform 403000 runs with a repetition rate of 1Hz takes about 112 hours. The latter provides an upper bound on the timescale of the proposed experiment to get a p-value of 1%. A similar analysis for a p-value of 10% shows that 46 hours are likely to be enough to detect the non-classical nature of a single photon superposed with vacuum using the human eye.

## 6 Conclusion

We have presented a concrete proposal for a quantum experiment with the human eye, including the full analysis of the measurement statistics. It uses simple components, namely path-entanglement, displacement operations in phase space and non photon number resolving detectors, to certify with naked eyes the non-classical nature of a state of light.

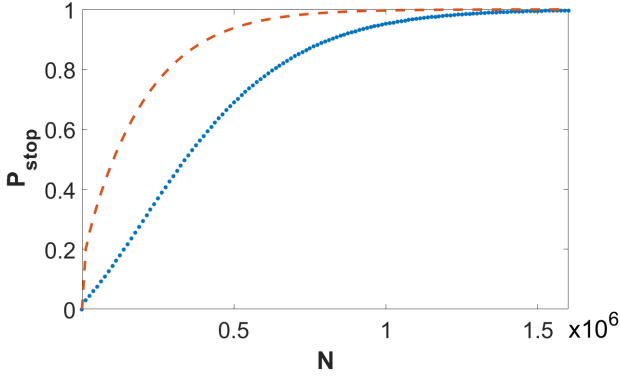


Figure 4: Probability to get enough statistics to conclude about non-classicality as a function of the number of runs  $N$  for a p-value of 1% (blue dotted line) and 10% (red dashed line).

We have given a detailed recipe using parametric conversions and photon counting techniques only, *i.e.* commercially available devices working at room temperature that are routinely used in practice. We have shown that the statistics obtained in a few tens of hours would be sufficient to certify non-classicality with a p-value of 10%. This was obtained with a realistic model of the human eye and taking loss and non-unit efficiencies of photon detectors into account. Following the implementation in Ref. [22], where a single photon and a coherent state with different polarizations are impinged on a polarization beamsplitter to follow the same optical paths and where a set of wave plates and a polarization beamsplitter are used to make the displacement operations, the setup can be made extremely stable without active stabilization of relative path-length fluctuations. It is thus very likely that the data acquisition can be stopped and started again later for several tens of hours without problem. We thus see our work as a ready-to-use proposal to realize the first experiment where the non-classical nature of light is detected with the human eye.

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## 8 Appendices

### 8.1 Autocorrelation with different arbitrary detectors

Let us recall the definition of a classical state as given in the main text:  $\rho_{cl} = \int d^2\alpha p(\alpha)|\alpha\rangle\langle\alpha|$  with  $p(\alpha) \geq 0$ . We now relax the constraint on the symmetry between the two arms in the autocorrelation measurement, and label (1,2) respectively the reflected and transmitted beams. Each of those beams is sent to a detector which can be different from the other one and the beamsplitter prior to detection is allowed to be unbalanced with coefficients  $r/t$ . Using the transformation rules for a coherent state on a beamsplitter, it is straightforward to express the probabilities of interest as an integral of the probabilities of singles for appropriate coherent states

$$\begin{aligned} P_{s_1}(\rho_{cl}) &= \int p(\alpha) P_{s_1}(\sqrt{r}\alpha) d^2\alpha \\ P_{s_2}(\rho_{cl}) &= \int p(\alpha) P_{s_2}(\sqrt{t}\alpha) d^2\alpha \\ P_c(\rho_{cl}) &= \int p(\alpha) P_{s_1}(\sqrt{r}\alpha) P_{s_2}(\sqrt{t}\alpha) d^2\alpha. \end{aligned}$$

Instead of the autocorrelation which is a ratio of two quantities, we focus on the difference

$$\begin{aligned} D(\rho_{cl}) &= P_c(\rho_{cl}) - P_{s_1}(\rho_{cl})P_{s_2}(\rho_{cl}) \\ &= \int p(\alpha) P_{s_1}(\sqrt{r}\alpha) P_{s_2}(\sqrt{t}\alpha) d^2\alpha \\ &\quad - \int p(\alpha) P_{s_1}(\sqrt{r}\alpha) d^2\alpha \int p(\alpha) P_{s_2}(\sqrt{t}\alpha) d^2\alpha, \end{aligned}$$

Note that  $D < 0$  implies  $g^{(2)}(0) < 1$ . Upon inserting  $\int p(\beta) d^2\beta = 1$  in  $P_c(\rho_{cl})$  and relabeling the dummy variable  $\alpha \leftrightarrow \beta$  in some of the terms we get

$$\begin{aligned} D(\rho_{cl}) &= \frac{1}{2} \int d^2\alpha p(\alpha) \int d^2\beta p(\beta) (P_{s_1}(\sqrt{r}\alpha) - P_{s_1}(\sqrt{r}\beta)) \\ &\quad \times (P_{s_2}(\sqrt{t}\alpha) - P_{s_2}(\sqrt{t}\beta)) \end{aligned}$$

We thus obtain that if the functions  $P_{s_{1/2}}(\alpha)$  are increasing with  $|\alpha|^2$ , then  $D(\rho_{cl}) \geq 0 \Leftrightarrow g^{(2)}(0)_{\rho_{cl}} \geq 1$ , which entails the validity of our witness even in the non-symmetrical case.

### 8.2 On the estimation of the average number of runs

We introduce a formalism to deal with the issue of finding a proper probability distribution for the number of runs. We write the sequence of measurements as a list of zeros and ones, binary stochastic results corresponding respectively to  $\chi_{mes} > \chi_0(N)$  and  $\chi_{mes} \leq \chi_0(N)$ . It illustrates the situation where an experimenter computes  $\chi$  after each measurement (or alternatively after each set of  $m$  measurements)

and decides if he carries on with the measures (“0”) or stops because the results are already satisfactory (“1”). Ideally what we would like to have is the probability  $P(n) = P(\underbrace{0, 0, \dots, 1}_n)$  to reach the required statis-

tics after exactly  $n$  runs. Unfortunately, obtaining this “true” probability numerically represents a computational challenge. What we output from our simulation  $P_{\text{stop}}(N)$  is the probability to get a one at  $N^{\text{th}}$  position regardless of the preceding sequence. Let’s compare the “cumulative distributions”

$$\begin{aligned} \sum_{n \leq N} P(n) &= P(1) + P(0, 1) + \dots + P(\underbrace{0, \dots, 0, 1}_{N-1}) \\ P(1) &= \sum_{i_2, \dots, i_N} P(1, i_2, \dots, i_N) \quad \text{where } i_k \in \{0, 1\} \\ &= P_{\text{stop}}(N) - \sum_{i_2, \dots, i_{N-1}} P(0, i_2, \dots, i_{N-1}, 1) \\ &\quad + \sum_{i_2, \dots, i_{N-1}} P(1, i_2, \dots, i_{N-1}, 0) \\ P(0, 1) &= \sum_{i_2, \dots, i_{N-1}} P(0, i_2, \dots, i_{N-1}, 1) \\ &= - \sum_{i_3, \dots, i_{N-1}} P(0, 0, i_3, \dots, i_{N-1}, 1) \\ &\quad + \sum_{i_3, \dots, i_{N-1}} P(0, 1, i_3, \dots, i_{N-1}, 0) \\ &\quad \vdots \\ \sum_{n \leq N} P(n) &= P_{\text{stop}}(N) + \sum_{n=0}^{N-2} P(\underbrace{0, \dots, 0, 1}_{n}, i_{n+2}, \dots, i_{N-1}, 1). \end{aligned}$$

Therefore  $P_{\text{stop}}(N) \leq \sum_{n \leq N} P(n)$ . We would like to translate it into an information on the expectation values. Let us switch to a continuous viewpoint and introduce functions  $f$  and  $g$  standing for the cumulative distributions, with  $\forall x \ g(x) < f(x)$  (thus  $g$  and  $f$  replace the  $P_{\text{stop}}$  and  $\sum_{n \leq N} P(n)$  of the previous paragraph). We write the expectation values difference and integrate by part

$$\begin{aligned} \int_0^M x f'(x) dx - \int_0^M x g'(x) dx &= M[f(M) - g(M)] \\ &\quad - \int_0^M \underbrace{(f(x) - g(x))}_{>0} dx. \end{aligned}$$

We need to know how the first term behaves when  $M \rightarrow \infty$ . We haven’t find a rigorous way to prove that it vanishes but we notice  $M[f(M) - g(M)] < M[1 - g(M)]$ , which we reasonably assume stays finite based upon the numerical simulations. The latter indeed reveals that  $N \mapsto N(1 - P_{\text{stop}}(N))$  shows a decreasing tendency after a given  $N$ . From this we deduce  $\langle N \rangle \leq \sum_n n \frac{dP_{\text{stop}}}{dn}$ .

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